Robust WLAN Device-free Passive Motion Detection

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Abstract-WLAN Device-free Passive (DfP) localization is an emerging technology that uses the widely deployed WiFi networks for detecting and localizing human presence within indoor environments. This paper presents an accurate and low-overhead technique for detecting human presence based on non-parametric statistical anomaly detection. This technique constructs profiles capturing the signal strength characteristics when no human is present within the area of interest and uses these profiles to identify any anomalies in the signal strength due to human motion activity. To adapt to changes in the environment, the constructed profiles are regularly updated by signal strength readings with low anomaly probability. Exponential smoothing is then used to reduce the effect of noisy readings in order to enhance the detection accuracy. Our work proved to be more robust and accurate than other DfP detection techniques, achieving a high detection accuracy of 4.7% miss detection rate and 3.8% false alarm rate, while requiring minimal deployment overhead.

Index Terms—Anomaly detection, device-free passive localization, motion detection, robust device-free localization.

I. INTRODUCTION

The increasing need for context-aware information and the rapid advancements in communication networks have motivated significant research effort in the area of location-based services. This effort resulted in the development of many location determination systems, including the GPS system [1], infrared-based (IR) systems [2], and radio frequency-based (RF) [3] systems. Moreover, motion detection systems, that aim at detecting the motion of an entity carrying a device, were also developed [4]–[8]. These systems require the tracked entity to carry a device that participates in the localization process. Thus, we refer to them as device-based systems.

Recently, we introduced the concept of WLAN Device-free Passive (DfP) localization [9], which takes advantage of the widely deployed WiFi networks to achieve its objectives. This concept was shown to be applicable for both detection and tracking of human entities [9]-[14]. DfP techniques neither require the tracked entity to carry a device nor participate in the localization process. This concept depends on the fact that the presence and motion of human entities in an RF environment affect the RF signal strength, especially when dealing with the 2.4 GHz band which is used in different IEEE standards such as 802.11b and 802.11g. DfP systems provide software only solutions that could be deployed on any available WiFi network, enabling a large set of applications including smart homes, intrusion detection, and border protection.

A typical WLAN DfP system (Figure 1) consists of signal transmitters, such as access points (APs), signal receivers or monitoring points (MPs), such as standard laptops, and an application server which collects and processes information about the received signals from each MP. The application server uses the collected information to perform the detection or tracking functions and initiates actions as needed.

WLAN DfP systems are mainly motivated by the observation that the current technologies which can provide devicefree tracking and detection (e.g. cameras [15], IR sensors, radio tomographic imaging [16], pressure sensors [17], etc) share the requirement of installing special hardware. The cost of such requirement might be prohibitive for homes and small businesses especially in some cases like cameras and IR sensors, whose functionality is limited to line-of-sight, and may require a high density installation to cover all site areas. In addition, regular cameras may fail to work in the dark or in the presence of smoke, and they can cause privacy concerns. On the other hand, RF signal propagation does not require lineof-sight for operation, and does not cause privacy concerns.

In this paper, we introduce a novel low-overhead WLAN DfP technique for detecting human presence in real wireless environments. This technique uses non-parametric statistical anomaly detection for its operation. It constructs profiles for the signal strength only when there is no human activity inside the area of interest in a short training phase, and then uses those profiles for human motion detection. We also introduce mechanisms for enhancing the detection accuracy by ensuring the robustness against the changes in the environment (e.g. humidity and temperature changes) that may cause deviations in the signal strength distributions. Furthermore, we present a mechanism for reducing the negative impact of noisy readings on the accuracy.

The main motivation for our technique is the strong assumptions made by previously proposed DfP detection techniques [9], [10]. For example, our previous work in [10] requires the construction of a human motion profile which assumes having access to all parts of the area of interest and also requires several hours of calibration even for a relatively small setup. Also, the techniques in [9], [10] were evaluated in



Fig. 1. An example of a typical device-free passive system deployed in a typical environment.

controlled environments or in small-scale real environments. Finally, earlier DfP detection techniques do not provide any mechanisms to adapt to changes in the environment, and their performance may degrade in real environments due to the dynamic changes in the signal strength distributions.

This paper is organized as follows: The details of the statistical anomaly detection technique is presented in Section II. Then, in in Section III, we present a mechanism for updating the constructed profiles in order to achieve robustness by adapting to changes in the environment. Additionally, due to noisy signal readings, false alarms of human presence can occur, therefore we present a technique to reduce the false alarm rate in Section IV. In Section V, we evaluate the proposed technique in a large-scale real environment rich in multi-path and show how our work outperforms the state-of-the-art DfP detection techniques. Finally, we conclude the paper and discuss future work directions in Section VI.

II. ANOMALY-BASED DfP MOTION DETECTION

In this section, we introduce our statistical anomaly-based *DfP* motion detection technique. Our proposed approach works in two phases:

- 1) *Training or offline Phase:* a short period of few minutes during which a silence or normal profile is constructed for each stream. The silence profiles capture the behavior of the signal strength readings when the area of interest has no human presence within a short window of time leading to minimal deployment overhead. We give the details of that phase in Section II-B
- 2) Monitoring or online Phase: In this phase, the signal readings received at the monitoring points are compared against the constructed silence profiles to detect any anomalous behavior (human presence). We give the details of that phase in Section II-C

We start by laying out the mathematical framework for the approach then give the details of the two phases.

A. Mathematical Notations

Let $s_{j,t}$ denote the received signal strength reading for a stream j at a time instant t. Our technique considers the



(c) Sample variance silence profile (d) Sample variance silence profile two weeks later

Fig. 2. Comparison between the sample mean and sample variance silence profiles showing the robustness of the latter. Subfigures (a) and (b) show two-week separated sample mean silence profiles for a wireless stream, while subfigures (c) and (d) show two-week separated sample variance silence profiles for the same wireless stream.

behavior of a sliding window $W_{j,t}$ of size l that ends at time t, i.e. $W_{j,t} = [s_{j,t-l+1}, s_{j,t-l+2}, ..., s_{j,t}]$. Each sliding window $W_{j,t}$ is mapped to a single feature or value $x_{j,t}$. The feature we choose here is the sample variance. Thus, $x_{j,t} = \frac{\sum_{i=1}^{l} (s_{j,t-l+i} - \bar{s}_{j,t})^2}{l-1}$, where $\bar{s}_{j,t}$ is the mean signal strength of $W_{j,t}^{-1}$. From another perspective, the variance is a relative measure as it measures difference about the mean. This means that sample variance profiles will be less affected by the temporal variation shifts that occur in the signal strength histograms, as compared to sample mean profiles (Figure 2). This implies that the sample variance profiles will be more robust than sample mean profiles.

B. Capturing Signal Behavior in Silence

During the offline phase, a normal profile is constructed for each stream independently. The normal profiles comprise the signal strength variance characteristics during the silence period, i.e. when there is no human in the area of interest. In order to construct a normal profile for a given wireless stream, our technique extracts the sample variance values of a sliding window over the collected normal signal strength readings for that stream. Then, the distribution of the extracted variance values is estimated using non-parametric kernel density estimation. These estimated densities are used during the online phase as a reference for statistical anomaly detection.

Formally, for a stream j, given a set of n sliding windows, each of length l samples, each window $W_{j,i}$ is mapped to a value $x_{j,i}$ as mentioned earlier. Assume f_j is the density function representing the distribution of the observed $x_{j,i}$'s. Then, given a random sample $x_{j,1}, x_{j,2}, ..., x_{j,n}$, the estimated

 $^{^{1}}$ In [18], we show experimentally that the sample variance as a dispersion measure, can be better used to identify the signal strength changes due to human activity.

density function \hat{f}_j is given by [19]:

$$\hat{f}_j(x) = \frac{1}{nh_j} \sum_{i=1}^n K\left(\frac{x - x_{j,i}}{h_j}\right) \tag{1}$$

where h_j is the bandwidth and K is the kernel function. The choice of the kernel function is not significant for the results of the approximation [20]. Thus, we chose the the Epanechnikov kernel as it is bounded and efficient to integrate, and used Scott's rule to estimate the optimal bandwidth [20].

C. Anomalies Detection: Human Presence Detection

We now consider the online phase during which the anomalies in the signal strength are detected. Anomalies in the signal strength occur due to signal fluctuations caused by people movement. The normal profiles, constructed during the offline phase, are used as a reference to detect those anomalies. In particular, for a sliding window $W_{j,t}$ for a stream j at a given time instant t, the sample variance value $x_{j,t}$ is calculated. A stream j is considered anomalous if $x_{j,t}$ is above a critical bound u_j . Given a significance parameter α and assuming \hat{F}_j is the CDF of distribution shown in Equation 1, the upper bound u_j will be equal to the $100(1 - \alpha)^{th}$ percentile of the CDF function, such that $u_j = \hat{F}_j^{-1}(1 - \alpha)$.

To quantify the significance of any anomalous activity, an anomaly score $a_{j,t}$ is calculated for each stream j. For a given window, $W_{j,t}$, the anomaly score, $a_{j,t}$, can be calculated as: $a_{j,t} = \frac{x_{j,t}}{u_j}$ where $x_{j,t}$ is the sample variance of the current sliding window and u_j is the critical value. This means that a detected anomalous sliding window will have an anomaly score greater than one and a silence sliding window will have an anomaly score of less than one. This anomaly score will be used in the next two sections for enhancing the detection accuracy and ensuring the robustness of the used normal profiles.

It should be noted that the parameters window size (l) and significance (α) need to be tweaked to control the accuracy of the statistical anomaly-based motion detection technique. Analysis of these two parameters is presented in Section V-C.

In its current form, the performance of the proposed anomaly-based technique can get affected by noisy readings and by the dynamic changes in signal strength distributions. Therefore, in the next two sections, we present enhancements to our technique. Mainly, we provide a mechanism for updating the constructed silence profiles automatically during the monitoring phase to increase the robustness of those profiles. In addition, we present a technique for reducing the negative effect of the noisy readings on the detection accuracy.

III. ADAPTING TO CHANGES IN THE ENVIRONMENT

Figure 2 shows that the sample variance profiles will be more robust than the sample mean profiles. However, the constructed sample variance profiles are still subject to signal strength variations due to the dynamic changes in the wireless environments and hence, the stored profiles may not capture the true silence state. Therefore, the silence profiles captured during the offline phase need to be updated continuously during the online phase in order to adapt to any possible changes in the signal strength distributions.

The technique we propose for updating the normal profiles in the online phase is based on the automatic update of the estimated density in Equation 1. This is handled by adding $x_{j,t}$'s that do not have high anomaly scores in average to the samples used for estimating the density. In particular, during the online phase, we group the consecutive $x_{j,t}$'s in disjoint groups of size l_{update} . The group that has an average anomaly score of less than one is added to the normal profile. The parameter l_{update} can be tuned to provide the desired performance. We discuss the effect of the l_{update} parameter in detail in Section V-C2.

In order to give higher priority to the new data added to the normal profiles, we give higher weight to the recent samples in Equation 1 instead of assuming equal weights. Therefore, Equation 1 is modified to:

$$\hat{f}_j(x) = \frac{1}{h_j} \sum_{i=1}^n w_i K\left(\frac{x - x_{j,i}}{h_j}\right)$$
 (2)

where $\sum_{i=1}^{n} w_i = 1$. We choose linear weights such that $w_i = \frac{i}{n(n+1)/2}$ (*n* is constant). We found that exponential weights do not provide good performance due to the high discrimination introduced between older and newer data.

IV. HANDLING NOISY READINGS

Due to the noisy nature of wireless environments, anomaly detection techniques operating on wireless signals can generate false alarms if the alarms were only generated due to the changes in a single stream. Therefore, a mechanism is needed to reduce those false alarms in order to enhance the detection performance. The technique we employ for the reduction of false alarms is based on fusing the information acquired from each wireless stream.

Since we quantify each event by an anomaly score, these scores can be used to enhance the detection performance. To achieve that, a global anomaly score a_t is calculated by summing the individual anomaly scores for each stream. If a noticeable change in a_t occurs, based on a threshold, while at least one stream is anomalous, this implies the start of an anomalous behavior. Exponential smoothing is used to reduce the noisy samples effect on the a_t curve, and hence reduce the false alarm rate. This technique makes use of the history of the activity state inside the environment through the usage of exponential smoothing. It also makes use of the locality of human motion, meaning that the human will continue to affect the same stream and/or other streams near it, causing the sum of anomaly scores smoothed curve to have higher values during the motion period.

V. EXPERIMENTAL EVALUATION

In this section, we present the evaluation of our proposed technique showing that it satisfies the design goals of being low-overhead, accurate, and robust against the changes in the



Fig. 3. Testbed layout and motion pattern.

environment. Also, a rationale for choosing values for the window size l, the significance α and the update window size l_{update} is presented. Finally, a comparison with previous WLAN *DfP* detection [9], [10] techniques is provided to illustrate the superiority of our proposed technique.

A. Experimental Testbed and Data Collection

The experiments were conducted in an office of approximately 2000 ft^2 covered with typical furniture (Figure 3). Two sets of data were collected two weeks apart to evaluate the robustness of the proposed technique. We used four Cisco Aironet 1130AG series access points and three DELL laptops equipped with D-Link AirPlus G+ DWL-650+ Wireless NICs.

For the data collection, sets of normal (silence) state readings and continuous human motion readings were collected. A total of about one hour and 15 minutes of data was collected; this includes three motion sets each lasting approximately three minutes. A motion set covers the entire area of the testbed, as shown by the red line in Figure 3, and represents the motion of a single person moving around the site continuously without any stops. For the evaluation, the training period was chosen to be the first two minutes only of the entire data collected during the absence of human motion. In addition, only one person moved in the area of interest.

B. Evaluation Metrics

We use two main metrics to analyze the detection capability: the false positive (FP) rate and the false negative (FN) rate. The false positive rate is the rate at which false alarms are generated while there is no human motion in the area of interest. The false negative rate refers to the probability of miss detection (i.e. when the technique fails to detect human motion in *any place* in the area). We also use the F-measure later when we compare the performance of the proposed technique to other DfP techniques.

It should be noted that each anomalous event may not be detected simultaneously after its occurrence. Therefore, we



Fig. 4. The Anomaly-based technique performance showing the effect of the enhancements

also study the detection latency, i.e. how much time is needed to associate an anomalous sample with a detected event. The overall 90^{th} detection latency was found to be one second at most. Due to space constraints, we report the accuracy results only.

C. Detection Performance

Figure 4 summarizes the performance of the proposed technique. It also illustrates how the enhancements discussed in Sections III and IV enhance the overall accuracy. In the following sections, we analyze the effect of the parameters on the detection performance.

1) Anomaly-Based Motion Detection: As mentioned earlier, the sliding window size l and the significance α need to be tweaked to get the desired detection performance. Figure 5 illustrates the effect of these parameters on the detection performance. The figure shows that choosing a too short window size will make the detection technique less sensitive to human motion. On the other hand, choosing a very large window size will introduce a very high FP rate. For the significance parameter, as α decreases, the FP rate decreases and the FN rate slightly increases. This means that increasing the significance will result in less sensitivity. Therefore, to balance the different performance metrics, we choose l = 5and $\alpha = 0.01$. From Figure 4, it can be noted that the FP rate of the basic anomaly-based technique without the enhancements is high. This is mainly because the two-minute training period is not sufficient for handling one hour of operation due to the possible changes in the environment, therefore the profile adaptation mechanism is needed along with noise handling.

2) Capturing Changes in the Environment: The update window size l_{update} is the key parameter in adapting to changes in the environment. Choosing a too small l_{update} will result into high sensitivity to noisy readings causing a high FP rate. On the other hand, a very large l_{update} will make the technique less sensitive to human motion causing a higher FN rate. Figure 6 illustrates the effect of the update window size on the detection performance when l = 5 and $\alpha = 0.01$. The figure shows that an update window size between 10 and 20 is sufficient to





(b) False Positives Rate

Fig. 5. The effect of the window size l and the significance α on the detection performance



Fig. 6. Effect of the update window size parameter (l_{update}) .

reduce the high FP rate without causing much increase to the FN Rate. Thus, we choose $l_{update} = 15$.

3) Noisy Readings Handling: By fusing the statuses of all streams, the false alarm rate can be reduced by generating alarms only when there is a high probability that there is human motion inside the area. The technique described earlier in Section IV mainly studies a smoothed curve of the sum of anomaly scores in order to avoid any sudden noise that may occur in the streams. Figure 7 displays the sum of anomaly scores curve for the data of our experiment. To reduce the FP rate, the curve is exponentially smoothed with a smoothing coefficient of 0.04. To declare an alarm, a large increment in the smoothed



Fig. 7. Curves representing the values of the sum of anomaly scores before and after exponential smoothing throughout the experiment.



Fig. 8. Performance comparison with previous DfP detection techniques.

curve, by more than 20% to 25% from the normal curve level, is required. According to our experience with the technique, deviations from these parameters values will not lead to significant degradation in the detection performance. The figure shows that the motion periods are clearly distinguishable from the silence state. Figure 4 shows the enhancement achieved by adding the noise handling mechanism. It should be noted that this technique was also able to reduce the FN rate, as some of the previously undetected events could be detected now because this technique makes use of the history of the state of the activity inside the area of interest as described earlier.

In summary, our experimental evaluations showed that it is clearly necessary to include the profile update and noise handling mechanisms to achieve high accuracy in real environments. Figure 4 shows that the proposed technique, with its enhancements, can achieve a high detection accuracy reaching 4.7% miss detection rate and 3.8% false alarm rate.

D. Comparison with DfP Detection Techniques

In this section, we compare the performance of our statistical anomaly-based technique to the previous techniques devised for WLAN *DfP* detection: the moving average and moving variance techniques proposed in [9], and the maximum likelihood estimation (MLE) technique proposed in [10]. We consider two cases for the comparison: The first is when the techniques are tested with the same data sets that were used to train them (if any). This is to test the best attainable accuracy. The second case is when the testing data set is collected two weeks after the data sets used for training. This is to test the robustness of each technique to changes in the environment. Figure 8 shows the comparison results in terms of the Fmeasure in both cases.

According to the figure, our proposed technique is better than all the other techniques in both cases. Furthermore, the enhancement is more clear in the second case. This is due to the robustness of the profiles that our technique uses, as it uses the sample variance for its operation in addition to employing techniques for adapting these profiles to the changes in the environment. The figure also shows that the performance of the MLE technique is the least in the second case as it uses the sample mean signal strength values as the feature used for classification. Therefore, after two weeks, the distribution of the signal strength is expected to deviate from the learned one. This is consistent with the profiles comparison provided before in Figure 2. It also should be noted that the moving average technique does not store any profiles. Therefore, its F-measure is relatively low but almost the same in both cases.

From the overhead perspective, the moving average technique has no profile construction overhead as it does not require any learning phase. Also, the moving variance and our anomaly-based techniques require the construction of normal profiles by collecting samples for two minutes when the human is not present. On the other hand, the maximum likelihood classification technique has the worst overhead as it requires the construction of a motion profile at each location in the area of interest in addition to the normal profile.

Therefore, in terms of both the detection accuracy and the deployment overhead, the proposed anomaly-based detection technique does provide a better practical technique than the other *DfP* techniques, since it has the highest accuracy and robustness, and requires minimal overhead.

VI. CONCLUSIONS AND FUTURE WORK

In this paper, we presented a novel technique for device-free passive human motion detection using the already installed wireless networks. A non-parametric statistical anomaly detection techniques was employed to provide the detection capability. We also presented a technique for adapting to changes in the environment by capturing changes that occur in signal strength readings over time, hence enhancing the detection accuracy and improving its robustness. The proposed technique was evaluated in a real environment providing an accurate detection capability reaching a 4.7% miss detection rate and a 3.8% false alarm rate. The performance of the proposed techniques for WLAN *DfP* detection. The results showed that our work outperformed the previous techniques in terms of robustness and accuracy.

For future work, we plan to analyze the effect of using more signal features other than the signal variance. We also plan to compare the performance of our non-parametric statistical anomaly detection technique to a parametric one. Another direction is to integrate our work with the *DfP* tracking systems to enhance their tracking accuracy.

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